One Order/Production Opportunity

Module Four presented how the manufacturer can decide how many units of a seasonal product to produce for the upcoming season. However, it was assumed that there was no initial inventory. Here, that assumption is relaxed and the following example shows what happens.

Example

The initial inventory is 1,000 units, and the manufacturer is trying to decide whether to produce 2,000 additional units or zero additional units for the upcoming season.

The demand from the distribution center to the manufacturer could be any of the following with the given probabilities:

Demand from distribution center	Probability
1,000	0.25
2,000	0.35
3,000	0.15
4,000	0.25

The following information is also given:

- Fixed cost of production = $10,000
- Variable cost of production = $50 per unit
- Selling price = $150 per unit
- Salvage value of unsold products = $25 per unit

If the manufacturer is considering production quantities of 2,000 additional units or zero additional units, which of these two options should the manufacturer choose?

If 2,000 additional units are produced, regardless of the demand from the distribution center, the production cost incurred by the manufacturer will be fixed cost + variable cost = $10,000 + (2,000 * $50) = $110,000.

The revenue, however, may be different for different demands from the distribution center. For example, if the demand is 1,000 units, the manufacturer will sell 1,000 units to the distribution center, and the remaining 2,000 units will be salvaged. Revenue = (1,000 * $125) + (2,000 * $25) = $175,000. The profit will be Revenue - Cost = $175,000 - $110,000 = $65,000.

If the demand is 2,000 units, the manufacturer will sell 2,000 units to the distribution center, and the remaining 1,000 units will be salvaged. Revenue = (2,000 * $125) + (1,000 * $25) = $275,000. The profit will be Revenue - Cost = $275,000 - $110,000 = $165,000.

If the demand is 3,000 units or 4,000 units, the manufacturer will sell all of the 3,000 units on hand. Revenue = 3,000 * $125 = $375,000. The profit will be Revenue - Cost = $375,000 - $110,000 = $265,000.

Demand	Probability	Revenue	Profit
1,000	0.25	$175,000	$65,000
2,000	0.35	$275,000	$165,000
3,000	0.15	$375,000	$265,000
4,000	0.25	$375,000	$265,000

The expected profit of the manufacturer is the *sum-product* of the profits and the respective probabilities. In other words, expected profit if the manufacturer produces 2,000 additional units = (0.25 * $65,000) + (0.35 * $165,000) + (0.15 * $265,000) + (0.25 * $265,000) = $180,000.

If zero additional units are produced, regardless of the demand from the distribution center, the production cost incurred by the manufacturer will be $0.

In this case, the revenue too will be the same for different demands from the distribution center. For example, if the demand is 1,000 units or 2,000 units or 3,000 units or 4,000 units, the manufacturer will sell all of the 1,000 units on hand. Revenue = 1,000 * $125 = $125,000. The profit will be Revenue - Cost = $125,000 - $0 = $125,000.

Demand	Probability	Revenue	Profit
1,000	0.25	$125,000	$125,000
2,000	0.35	$125,000	$125,000
3,000	0.15	$125,000	$125,000
4,000	0.25	$125,000	$125,000

The expected profit of the manufacturer is the *sum-product* of the profits and the respective probabilities. In other words, expected profit for the production quantity of zero additional units = (0.25 * $125,000) + (0.35 * $125,000) + (0.15 * $125,000) + (0.25 * $125,000) = $125,000.

Since the expected profit is higher for production quantity of 2,000 additional units, the manufacturer should be recommended to produce 2,000 additional units for the season.

Multiple Order/Production Opportunities

The previous section considered a seasonal product with one order/production opportunity. This section now considers a generic product with multiple order/production opportunities. Like in the previous section, here too, the demand is uncertain and there is some initial inventory. A few definitions are in order:

- **Safety stock**: If a firm does not allow its inventory to fall below a certain level, that level is called the safety stock
- **Reorder level**: If a firm places an order for products if the inventory level falls to or below a certain level, that level is called the reorder level
- **Lead time**: It is the duration between placement of an order and receipt of that order
- **Service level**: The percentage of customers who get the product of their choice when they want it and where they want it; for example, if a retail store maintains a service level of 90%, the chance of stockout at that retail store is 10%

Using statistical concepts learned in QSO 510: Quantitative Analysis for Decision Making, the prerequisite of this course, the following formulae are derived. For the derivation process, refer to the textbook chapter cited in the Module Resources section:

Safety Stock = z * (Standard Deviation of Demand) * $\sqrt{\text{Lead Time}}$
Where z is obtained using the desired service level and normal distribution concepts from QSO 510. The example that follows the below formulae illustrates the concepts.

Reorder Level = (Average Demand * Lead Time) + (Safety Stock)

Recall from Module Four that the Economic Order Quantity $= Q^* = \sqrt{\frac{2RK}{h}}$. Recall also that slightly changing the value of Q* will not increase the total annual cost by a lot. Hence, even though the EOQ formula was derived for a constant demand situation, $Q = \sqrt{\frac{2RK}{h}}$ is used here as well. Notice that the asterisk is removed because Q is no longer considered the "best" order quantity, given the fact that demand is no longer constant. However, R (constant annual demand) in this formula is replaced by the Average Demand calculated using historical data.

Hence, $Q = \sqrt{\frac{2 * \text{Average Demand} * K}{h}}$

Recall from Module Two that the average inventory level is typically calculated by taking the average of the inventory level (Safety Stock) just before receiving the order and the inventory level (Safety Stock + Q) just after receiving the order.
Hence, the average inventory level is (Safety Stock + Safety Stock + Q) / 2
= (Q/2) + (Safety Stock).

The following example shows how all of the above formulae are applied:

Example

A retail store orders from a manufacturer. The ordering cost is $2,000 per order, and the holding cost is $1 per unit per week. Also, the lead time is two weeks, and the desired service level is 97%.

The following table provides historical demand data for the last 8 weeks:

Week	I	II	III	IV	V	VI	VII	VIII
Demand	100	145	125	184	200	98	118	142

Average Demand (using the AVERAGE function in MS Excel) = 139 units per week
Standard Deviation of demand (using the STDEV function in MS Excel) = 37 units per week.

Service level = 97%. This means, z = NORMSINV (0.97) = 1.88.

Recall from QSO 510 that if area to the left of X under a normal curve is given, X is calculated by first obtaining the value of z by using the NORMSINV function in MS Excel.

Safety Stock = z * (Standard Deviation of Demand) * $\sqrt{\text{Lead Time}}$

$$= 1.88 * 37 * \sqrt{2} = 98 \text{ units}$$

Reorder Level = (Average Demand * Lead Time) + (Safety Stock)

$$= (139 * 2) + (98) = 376 \text{ units}$$

$$\text{Order Quantity, } Q = \sqrt{\frac{2 * \text{Average Demand} * K}{h}} = \sqrt{\frac{2 * 139 * 2{,}000}{1}} = 746 \text{ units per order}$$

Average inventory level = (Q/2) + (Safety Stock) = (746 / 2) + (98) = 471 units